Kinematics/Mechanics

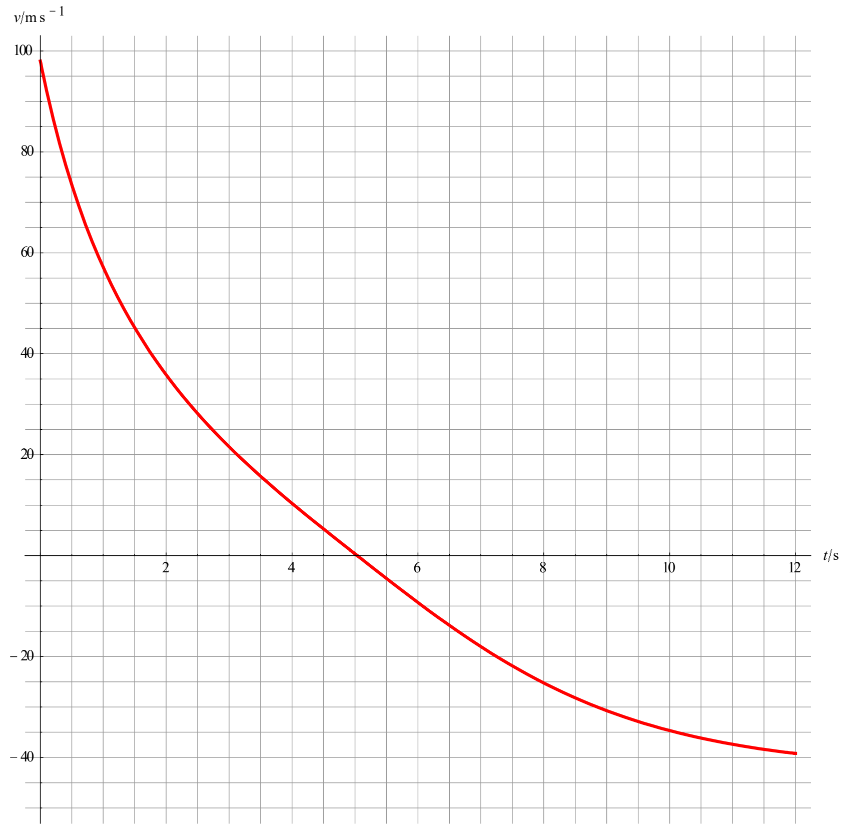
(a) A projectile in vacuum is launched vertically upwards with speed 98 m s^{-1}.

Determine

(i) the time taken to reach the maximum height,
(ii) the maximum height reached,
(iii) the speed at which the projectile impacts the ground.

(b) Draw graphs to show the variation with time of
(i) the velocity of the projectile,
(ii) the acceleration of the projectile,
(iii) the position of the projectile relative to the ground.

(c) The same projectile is now launched vertically upwards with speed 98 m s^{-1} but this time there is an air resistance force opposing the motion. The air resistance force is zero when the speed is zero and increases with increasing speed.
The graph shows the variation with time of the velocity of the projectile until just before it impacts the ground at, approximately, t = 12 s.



(i) Estimate the maximum height reached by this projectile.

(ii) Estimate the magnitude of the initial acceleration of the projectile.

(iii) State and explain the acceleration of the projectile at t = 5.0 s.

(iv) State the area under the graph between t = 5.0 s and t = 12 s.

(v) Estimate the average speed of the projectile on the way up and on the way down.

(vi) Estimate the magnitude of the average acceleration of the projectile on the way up, $|\bar{a}_{\text{up}}|$, and on the way down, $|\bar{a}_{\text{down}}|$.

(vii) Explain why $|\bar{a}_{\text{up}}| > |\bar{a}_{\text{down}}|$.

(viii) The graph appears straight for speeds under about 10 m s^{-1}. Suggest a reason for this.

(ix) Sketch a graph to show the variation with time of the acceleration. No numbers are required, just the general shape.

(d) Determine the mechanical energy of the projectile that was dissipated as thermal energy. The mass of the projectile is 6.0 kg.

(e) The resistance force is given by $F = kv^2$. Using data at t = 0, determine k.

Answers

(a)

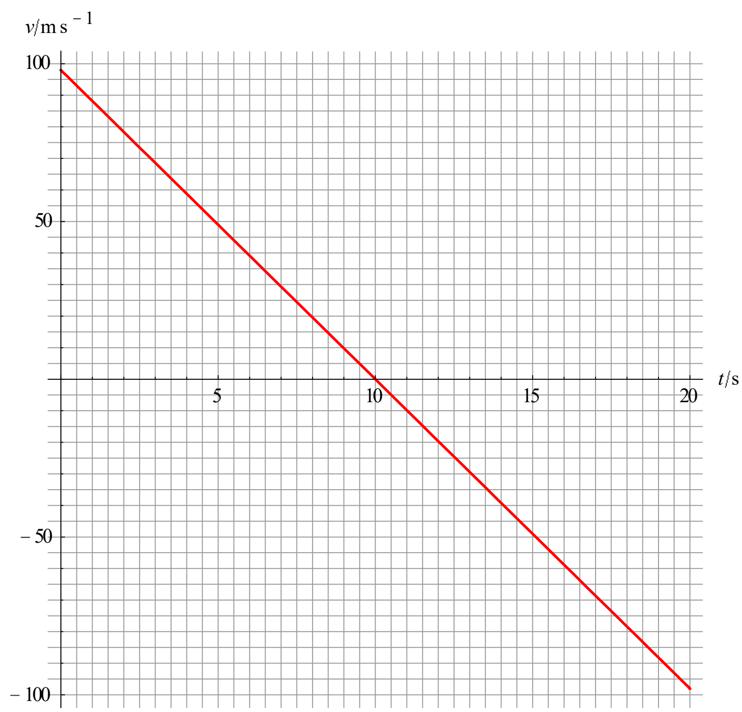
(i) $v = u - gt = 0 \quad \Rightarrow \quad 0 = 98 - 9.8t \quad \Rightarrow \quad t = 10\text{ s}$.

(ii) $y = ut - \frac{1}{2}gt^2 = 0 \quad \Rightarrow \quad y = 98 \times 10 - \frac{1}{2} \times 9.8 \times 10^2 \quad \Rightarrow \quad y = 490\text{ m}$.

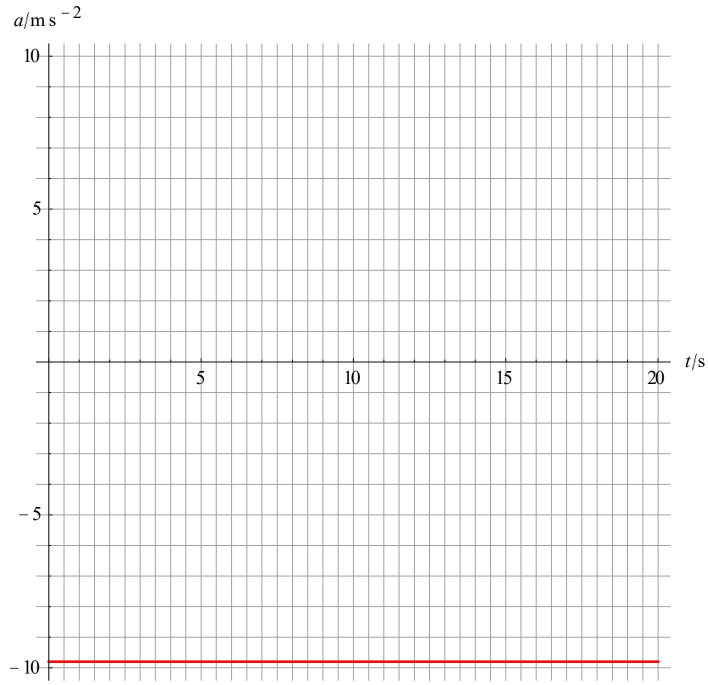
(iii) $v = u - gt \quad \Rightarrow \quad v = 98 - 9.8 \times 20 \quad \Rightarrow \quad v = -98\text{ m s}^{-1}$.

(b)

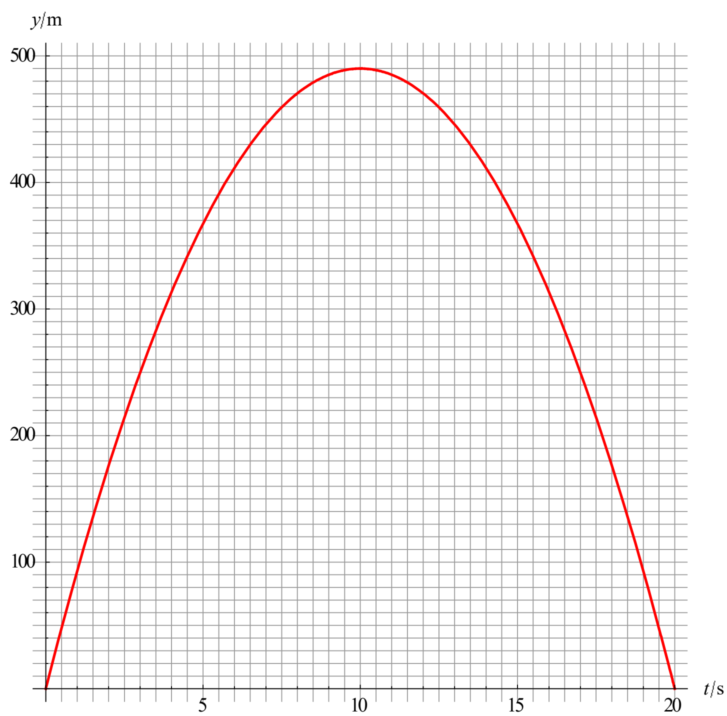
(i)



(ii)



(iii)



(c)

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(i) There are about 68 small squares under the graph and each square has area 2.5 m. So, the height is about 170 m.

(ii) We need to draw a tangent to the curve at $t = 0$ and find its gradient. It is about –61 m s^{-2} so the magnitude is 61 m s^{-2}.

(iii) It is –9.8 m s^{-2} because at this time the speed is zero and so the air resistance force is also zero. The only force on the projectile is its weight.

(iv) It is 170 m since this is the distance fallen which equals the distance raised.

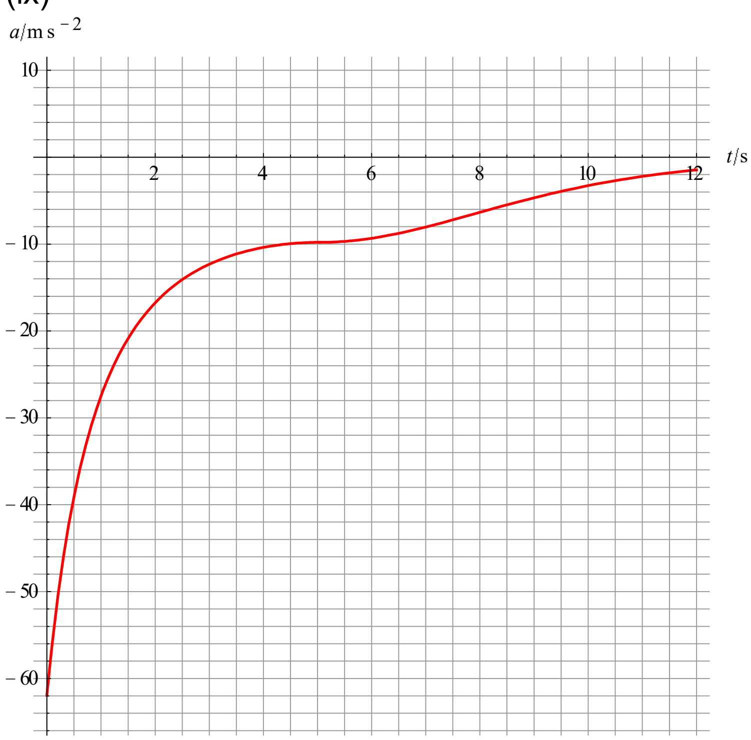
(v) $\bar{v}_{up} = \frac{170}{5.0} = 34 \text{ m s}^{-1}$; $\bar{v}_{down} = \frac{170}{12-5.0} \approx 24 \text{ m s}^{-1}$.

(vi) $|\bar{a}_{up}| = \frac{98}{5.0} \approx 20 \text{ m s}^{-2}$ and $|\bar{a}_{down}| = \frac{40}{12-5.0} = 5.7 \text{ m s}^{-2}$.

(vii) On the way up both the weight and the resistance force act opposite to the velocity resulting in a greater acceleration.

(viii) At small speeds the air resistance force is negligible and so the only force on the projectile is the weight leading to a constant acceleration (of g) and hence a straight graph for velocity vs time.

(ix)



(d) The initial mechanical energy is just kinetic energy

$\frac{1}{2}mv^2 = \frac{1}{2} \times 6.0 \times 98^2 = 2.88 \times 10^4$ J and the final is also kinetic

$\frac{1}{2}mv^2 = \frac{1}{2} \times 6.0 \times 40^2 = 4.8 \times 10^3$ J. The energy dissipated is then the difference i.e.

2.4×10^4 J.

(e) At $t = 0$, $mg + kv^2 = -ma$ hence $6.0 \times 9.8 + k \times 98^2 = -6.0 \times (-61)$. This gives $k \approx 3.3 \times 10^{-2}$ kg m^{-1}.